

Computation of Spacecraft Signal Raypath Trajectories Relative to the Sun

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A computer program (CTS 41B) used to determine the trajectory of a spacecraft signal raypath has been updated to increase its usefulness during solar conjunctions (CTS 41C). The closest point of approach of the raypath to the sun is projected onto the surface and the solar latitude and longitude calculated. A sample computation and plots are given for the 1976 Viking solar conjunction. Eventually it may be possible to predict the communication link performance degradation in the near sun region due to solar activity.

I. Introduction

A double-precision computer program, CTS 41B (Ref. 1), was recently developed to determine the trajectory of a spacecraft telemetry signal raypath relative to some body (e.g., the sun), with respect to which the spacecraft is undergoing a superior conjunction. Such a determination is necessary in order to assess the effects imposed on the telemetry carrier during the superior conjunction. The evaluation of relationships between the quality of the communication channel and particular features of the conjunction body is facilitated by the newest version of this trajectory program, CTS 41C. While CTS 41C has been developed specifically for solar conjunctions, the program could be adapted for conjunctions involving planets or other bodies.

It is theoretically expected that the telemetry signal from a spacecraft which is undergoing superior conjunction with the sun should be most strongly affected by the solar atmosphere in the vicinity of the point of closest approach of the signal

raypath to the sun. Measurements made during the superior conjunctions of several different spacecraft (Ref. 2-7) have indicated that such generally tends to be the case. The coronal features which are actually encountered by telemetry signals during the superior conjunctions of deep space probes with the sun are difficult to observe by other available techniques. Features in the chromosphere and photosphere, however, are considerably more susceptible to the standard observational techniques and may therefore provide the best basis for the establishment of relationships between particular solar phenomena and the effects measured in the coronal communication channel.

Because the solar plasma is extended more or less radially away from the sun in the developing solar wind, the photospheric and chromospheric features that are most favorably located with respect to the signal raypath can be identified by projecting the point of closest approach of the raypath to the sun radially onto the photosphere. A finite interval of time may be required for the surface properties to be communicated to

the coronal regions traversed by the signal raypath; hence the solar longitude of the projected point must be corrected to account for the solar rotation during this time interval. Because different types of solar phenomena may be communicated through the corona at different speeds, no attempt to estimate this effect has been included in CTS 41C; it is assumed that users of this program will make the corrections that are appropriate to their specific situations.

II. Input

The location of a point on the solar surface can be specified in terms of its coordinates in solar latitude and solar longitude. The solar longitude, in turn, is specified by the Carrington solar longitude and the Carrington solar rotation number. In order to correctly compute the coordinates of a point on the surface of the sun through the program CTS 41C, it is necessary to input the Carrington solar rotation number and the Carrington solar longitude of the central meridian of the sun on the first day of the conjunction interval, unless that day also happens to be the first day of the year. These values can be found in the *American Ephemeris and Nautical Almanac* and should be entered in the program as the constants NROTZZ and CSLZZ, respectively. If the first day of the conjunction interval is the first day of the year (i.e., January 1), the correct longitude and rotation number will be automatically computed without requiring these inputs. In default of these inputs, the solar coordinates will be computed incorrectly (unless the conjunction interval begins on January 1), but all of the other computations will be done correctly; hence users who are not interested in locating the projected point of closest approach on the solar surface need not concern themselves with these inputs.

III. Computations

The basic coordinate system used in the raypath trajectory computations is a heliocentric system referred to the ecliptic plane, in which

$\hat{X}_{ec} \equiv$ direction from sun to earth

$\hat{Z}_{ec} \equiv$ normal to ecliptic plane, positive to north

The Y-axis completes the right-handed system, and the X-Y plane is the ecliptic plane.

In this coordinate system the coordinates of the point of closest approach of the telemetry signal raypath to the sun are given by (Ref. 1)

$$X_B = R \sin (SEP)$$

$$Y_B = Y_P R_{EB} / R_P$$

$$Z_B = \frac{Z_P R_{EB}}{R_P}$$

where

(X_B, Y_B, Z_B) = coordinates of point of closest approach

R = raypath offset from sun

SEP = sun-earth-probe angle

R_{EB} = distance from earth to point of closest approach

R_P = earth-probe range

(X_P, Y_P, Z_P) = coordinates of probe

Locations on the solar surface are better specified in a heliocentric coordinate system referred to the solar equatorial plane, in which

$\hat{Z}_{eq} \equiv$ direction of solar rotational axis

$\hat{X}_{eq} \equiv$ in plane of central meridian

with the X-Y plane being the solar equatorial plane.

The transformation from heliocentric-ecliptic coordinates to heliocentric-equatorial coordinates can be accomplished by a sequence of simple coordinate system rotations. The heliocentric-ecliptic coordinates are first rotated about the Z_{ec} axis (normal to the ecliptic plane) by the angle between the sun-earth line and the ecliptic-equatorial node. This angle, ϕ_n , is equal to the difference between the longitude of the sun, which is provided in record 27 of the geocentric block of coordinates on the DPTRAJ save tape read by the program CTS 41C, and the longitude of the ecliptic-equatorial node = 73 deg, 40 min + 50.25 sec $\times T$, where T is the time in years

since 1850. This coordinate rotation is thus performed by the matrix operator

$$M_1 = \begin{pmatrix} \cos \phi_n & \sin \phi_n & 0 \\ -\sin \phi_n & \cos \phi_n & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The new X-axis then lies along the ecliptic-equatorial node and is common to both the ecliptic and solar equatorial planes. A rotation about this axis by the angle between the ecliptic and solar equatorial planes, 7.25 deg, is performed by the matrix operator

$$M_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 7.25 & \sin 7.25 \\ 0 & -\sin 7.25 & \cos 7.25 \end{pmatrix}$$

The new Z-axis then coincides with the solar rotational axis. A rotation about this axis by the angle between the sun-earth line and ecliptic-solar equatorial node, in the opposite sense to the original rotation, is performed by the matrix operator

$$M_3 = \begin{pmatrix} \cos \phi_n & -\sin \phi_n & 0 \\ \sin \phi_n & \cos \phi_n & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The final coordinate system then has the Z-axis in the direction of the solar rotational axis, with the X-axis in the plane of the central meridian and the X-Y plane is the solar equatorial plane.

The coordinates of the point of closest approach of the signal raypath to the sun in the solar equatorial coordinate system are then given by

$$\begin{pmatrix} X_B \\ Y_B \\ Z_{B/eq} \end{pmatrix} = M_3 M_2 M_1 \begin{pmatrix} X_B \\ Y_B \\ Z_{B/ec} \end{pmatrix}$$

The point of closest approach lies at solar latitude

$$\lambda_B = \arctan \left(\frac{Z_{B/eq}}{\sqrt{X_{B/eq}^2 + Y_{B/eq}^2}} \right)$$

and differs in solar longitude from the central meridian by the angle

$$\phi_B = \arctan \left(\frac{Y_{B/eq}}{X_{B/eq}} \right)$$

The Carrington solar longitude of the central meridian on the k th day of the conjunction interval is computed according to

$$\theta_{cm}(k) = CSLZZ - \sum_{i=1}^k 360 \left(\frac{1}{T_{SID}} - \frac{1}{365.25 R_{EAU}^2(i)} \right)$$

where

$CSLZZ$ = Carrington solar longitude of central meridian on first day of conjunction interval

T_{SID} = sidereal solar rotation period = 25.38 days

$R_{EAU}(i)$ = earth-sun range in astronomical units on i th day of the conjunction interval

In this computation, the Carrington solar longitude of the central meridian decreases from the initial longitude, $CSLZZ$, as a consequence of the solar rotation. Each time the longitude becomes negative the Carrington solar rotation number is increased by unity and the Carrington solar longitude is increased by 360 deg. The Carrington solar longitude of the point of closest approach of the signal raypath on the k th day of the conjunction interval is then

$$\theta_B(k) = \theta_{cm}(k) + \phi_B$$

It is convenient to define the fractional Carrington solar rotation number corresponding to the solar longitude ϕ_B as

$$\eta_{FSR} = \eta_{CSR} + \frac{\theta_B}{360 \text{ deg}}$$

where

η_{FSR} = fractional solar rotation number

η_{CSR} = integral Carrington solar rotation number.

In addition to computing the location of the projected point of closest approach on the solar surface, the program CTS 41C also computes several trajectory parameters that are of potential value to experimenters and which were not computed by the program CTS 41B. One of these is the apparent angle between the sun-probe line and the ecliptic plane, as seen from earth. This angle, PEL, is defined by the equation

$$PEL = \arctan (Z_A / Y_A)$$

where (Y_A, Z_A) = coordinates of the spacecraft relative to the sun, as described in Ref. 1.

The sign convention was chosen to make $0 < PEL < 180$ when the probe is north of the ecliptic plane, $-180 < PEL < 0$ when the probe is south of the ecliptic plane, $|PEL| < 90$ when the probe is west of the sun, and $90 < |PEL| < 180$ when the probe is east of the sun.

The program CTS 41C also computes the sun-earth-probe angle (SEP), the sun-probe-earth angle (SPE), and the daily rate of change of the sun-probe-earth angle (DSPE). This latter quantity is computed from the equation

$$DSPE(k) = \frac{SPE(k+1) - SPE(k-1)}{2}$$

IV. Output

In addition to all of the output formerly provided by the program CTS 41B (Ref. 1), the program CTS 41C also provides tabular listings of the Carrington solar longitude, solar latitude, and fractional Carrington solar rotational number of the projection on the solar surface of the point of closest

approach of the telemetry signal raypath to the sun. The Carrington solar longitude of the central meridian is included in this tabulation. Tabular listings of the apparent angle between the sun-probe-earth angle and the daily rate of change of the sun-probe-earth angle are also provided for each day of the conjunction interval.

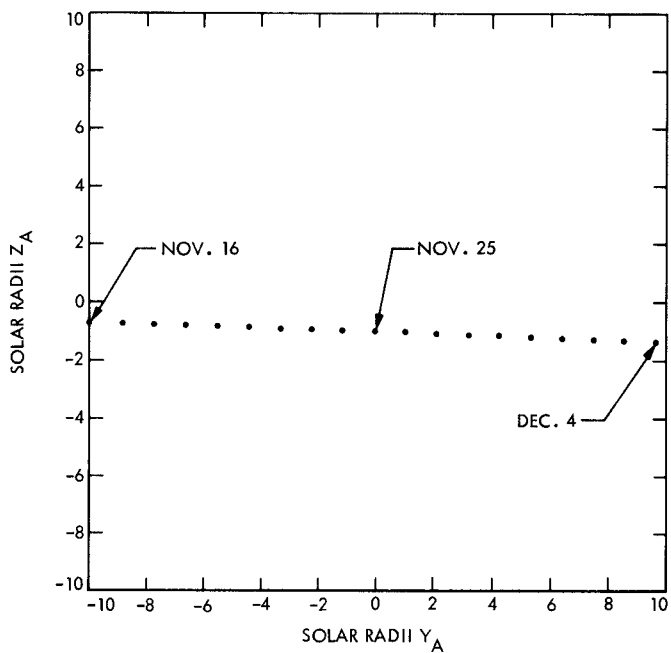
The coordinates of the projected point of closest approach are plotted in a format similar to that used in the standard synoptic charts of solar features. In this plot, the solar latitude of the point of closest approach is plotted between -90 and $+90$ deg, as usual. However, the solar longitude is plotted in terms of the fractional number of solar rotations within the conjunction interval, rather than the usual Carrington solar longitude. This choice was inspired by the constraints imposed by the standard computer plot routine. Each integral value on this scale corresponds to an integral solar rotation, at which point the Carrington solar longitude passes through $0/360$ deg. The initial Carrington solar rotation number is printed on the plot and corresponds to the first point plotted. The solar rotation numbers then increase from right to left, while the solar longitude within a given rotation increases from left to right. While this convention coincides with the standard convention used in synoptic charts of solar observations, the plotting device does not permit the solar longitude and solar rotation numbers to be printed directly on the plot; hence the method of fractional solar rotations was resorted to.

Sample plots made for the superior conjunction of the Viking spacecraft in 1976 are presented in Figs. 1 and 2.¹ Solar longitudes and solar rotation numbers have been added along the bottom of the computer plot (Fig. 2) to illustrate the manner in which they are related to the fractional rotation numbers printed by the computer. Several dates have also been added to the plots. These can be determined by referring to the printed output as described in Ref. 1.

¹ For this sample plot computed for the period Oct. 12 to Dec. 30, 1976 use for CTS 41C input data NROTZZ = 1647 and CSLZZ = 345.54 (see the 1976 *American Ephemeris and Nautical Almanac*, pages 359 and 357), for rotation number and value of L_0 , respectively, for Oct. 12, 1976.

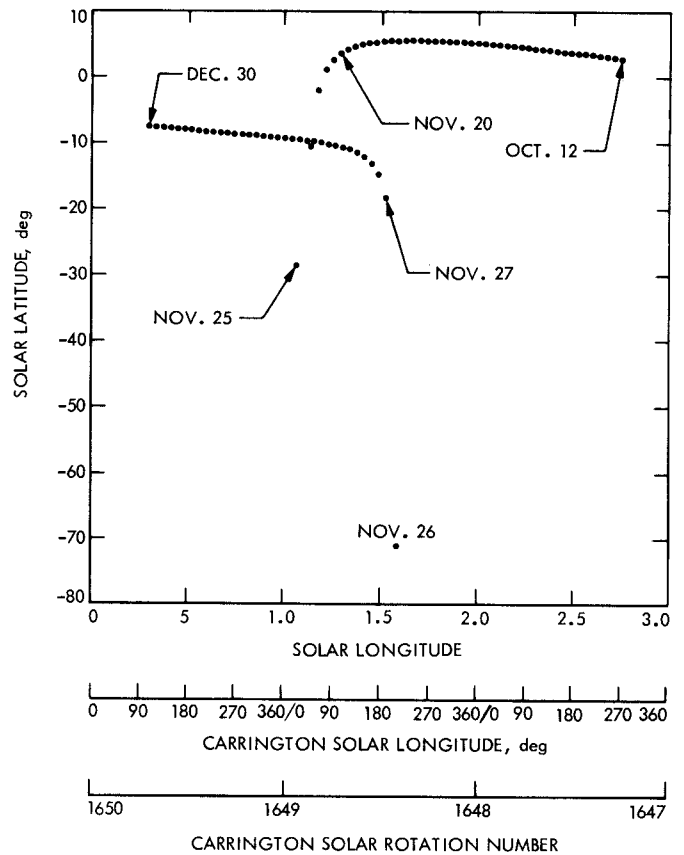
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DAY 1 DAY N
3.2100000+02 3.3900000+02

Fig. 1. Plot of the Viking spacecraft ray path offset from the sun as a function of 1976 data



BEG. ROT. No. YEAR
1.6470000+03 1.9760000+03

Fig. 2. Coordinates of the projected point of closest approach during the superior conjunction of the Viking spacecraft in 1976